460-2 International Economics

Lecture notes 3: Boom and bust with nominal wage rigidities

- Based on Schmitt-Grohe and Uribe (2013)
- Preferences
$$\mathrm{E}\left[\sum \beta^t U(c_t)\right]$$
  - Tradable and non-tradable goods
$$c_t = \left(c_t^T\right)^{\omega} \left(c_t^N\right)^{1-\omega}$$
- Endowment of tradables
$$y_t^T$$
  and production of non-tradables
$$y_t^N = f\left(h_t\right) = h_t^{\alpha}$$
- Inelastic supply of hours $\overline{h}$
$$h_t \leq \overline{h}$$
- We allow for unemployment $h_t < \overline{h}$ (see below)
- Budget constraint
$$P_t^T c_t^T + P_t^N c_t^N + (1 + r_{t-1}) E_t d_t = P_t^T y_t^T + W_t h_t + \Phi_t + E_t d_{t+1}$$
- E_t nominal exchange rate, P_t^T and P_t^N nominal prices
- Price of tradables pinned down at world price (normalized to 1) so
$$P_t^T = E_t$$
- Relative price of non-tradables
$$p_t \equiv \frac{P_t^N}{P_t^T} = \frac{P_t^N}{E_t}$$
- Demand for non-tradables
$$\frac{c_t^T}{c_t^N} = \frac{P_t^T}{P_t^N} = p_t$$
- Firms
$$\Phi_t = \max_{h_t} P_t^N f(h_t) - W_t h_t$$

- Optimality
$$\frac{W_t}{P_t^N} = f'(h_t) = f^{'}(f^{-1}(y_t^N))$$
which can be rewritten as
$$p_t = \frac{W_t/E_t}{f^{'}(f^{-1}(y_t^N))}$$

- Fixing values of c_t^T and W_t/E_t we can plot demand and supply of NT and find equilibrium value where
$$c_t^N = y_t^N$$

- Downward nominal wage rigidity:
  - if demand=supply at a y^N such that $f^{-1}(y^N) < \overline{h}$ then $h_t = f^{-1}(y^N)$ and wages remain at $W_t = W_{t-1}$
  - otherwise, wages increase up to the point where $f^{-1}(y^N) = \overline{h}$

- Assume pegged exchange rate
- Assume $U(c) = \log(c)$
- Then
$$\frac{1}{c_t^T} = (1+r_t)\beta \mathrm{E}_t \frac{1}{c_{t+1}^T}$$

- Temporary interest rate shock at 0
- Consumption
$$\begin{aligned} c_0^T &= y^T + d \\ c_t^T &= y^T - r\frac{1+r_0}{1+r}d \text{ for } t = 1, 2, ... \end{aligned}$$
Value of d determined by
$$\frac{1}{y^T + d} = \beta(1+r_0)\frac{1}{y^T - r\frac{1+r_0}{1+r}d}$$

- Social margins
- Maximize
$$\sum \beta^t U(c_t)$$

- Subject to
$$\frac{W_t}{f'(f^{-1}(c_t^N))} = \frac{c_t^T}{c_t^N}$$
and either
$$W_t = W_{t-1}$$
and
$$f^{-1}(c_t^N) \leq 1$$
or
$$f^{-1}(c_t^N) = 1$$
$$W_t \geq W_{t-1}$$
- Value function depends on state variables W_{t-1} and d_t
- Any time $c_t^T < c_{t-1}^T$ we have wage rigidity binding
- Optimality condition
- Higher W_0 has no effect on allocation at date 0, but tighten constraint at all future periods
- Reducing d reduces W_0, increases c_t^N for $t = 1, 2, ...$
- So condition is
$$\frac{\omega}{c_0^T} - \beta(1+r_0)\frac{\omega}{c^T} - \frac{\beta}{1-\beta}\frac{1-\omega}{c^N}\frac{dc^N}{dd}$$
- Deriving the last derivative
$$\frac{W_0}{f'(\overline{h})} = \frac{c_0^T}{f(\overline{h})}$$
$$\frac{W_0}{f'(h)} = \frac{c^T}{f(h)}$$
$$\frac{c_0^T}{c^T} = \frac{f(\overline{h})/f'(\overline{h})}{f(h)/f'(h)}$$
with Cobb-Douglas
$$\frac{c_0^T}{c^T} = \frac{\overline{h}}{h}$$
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